



EE 232 Lightwave Devices Lecture 4: Density of States, Quantum Wells and Wires

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Review of Quantum Mechanics

Schrodinger Equation

$$H\Psi(\vec{r},t) = i\hbar \frac{\partial}{\partial t} \Psi(\vec{r},t)$$

$$H = \frac{P^2}{2m} + V(\vec{r},t) \quad \text{Hamiltonian} = \text{Kinetic} + \text{Potential Energy}$$

$\Psi(\vec{r},t)$ Wavefunction

$$|\Psi(\vec{r},t)|^2 = \Psi(\vec{r},t) \cdot \Psi(\vec{r},t)^* \quad \text{Probability of finding particle at } \vec{r}$$

$\vec{P} = -i\hbar\nabla$ Momentum operator

$$\langle \vec{P} \rangle = -i\hbar \int \Psi(\vec{r},t)^* \nabla \Psi(\vec{r},t) d\vec{r} \quad \text{Average Momentum}$$

$$\langle \vec{r} \rangle = \int \Psi(\vec{r},t)^* \vec{r} \Psi(\vec{r},t) d\vec{r} \quad \text{Average Position}$$

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Electron Plane Wave

$$\Psi(\vec{r}, t) = e^{i\vec{k}\vec{r} - i\omega t}$$

$$\text{LHS: } H\Psi(\vec{r}, t) = \frac{\hbar^2 k^2}{2m} \Psi(\vec{r}, t) + V(\vec{r}, t) \Psi(\vec{r}, t)$$

$$P = \hbar k$$

$$\text{RHS: } i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t) = \hbar \omega \Psi(\vec{r}, t)$$

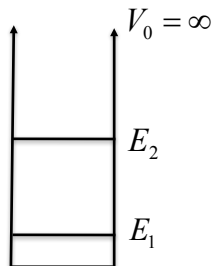
$$\Rightarrow \frac{\hbar^2 k^2}{2m} + V(\vec{r}, t) = \hbar \omega$$

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Example: Infinite Potential Well



$$x = 0 \quad x = L$$

Time Independent Potential

$$V(z) = \begin{cases} 0 & \text{for } 0 < z < L \\ \infty & \text{for } z < 0 \text{ or } z > L \end{cases}$$

$$\Psi(z, t) = \phi(z) e^{-i\omega t}$$

Solve Eigenvalue $E = \hbar \omega$ in

$$-\frac{\hbar^2}{2m} \frac{d^2}{dz^2} \phi(z) + V(z) \phi(z) = E \phi(z)$$

$$\text{For } 0 < z < L, \quad \frac{d^2}{dz^2} \phi(z) + \frac{2mE}{\hbar^2} \phi(z) = 0$$

$$\phi(z) = \begin{cases} \sin(kz) \\ \cos(kz) \end{cases}$$

$$\text{B.C. } \phi(z=0) = \phi(z=L) = 0$$

$$\phi_n(z) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} z\right)$$

$$E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{L}\right)^2$$

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Typical Examples

In GaAs, $m_e^* = 0.067m_0$

For a 10-nm-wide potential well ($L = 10\text{nm}$)

$$E_1 = 56 \text{ meV}$$

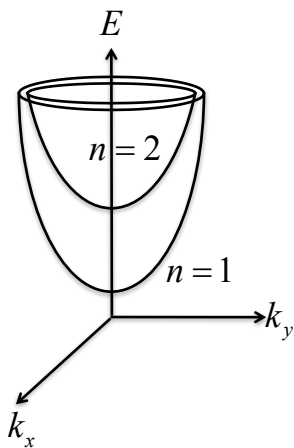
$$E_2 = 4E_1 = 224 \text{ meV}$$

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Complete Wavefunction for Infinite Potential Well



$$\Psi^r(r, t) = \phi^r(x, y)\phi(z)e^{-i\omega t}$$

Electron confined in z , but free in x, y

\Rightarrow Plane wave in x and y

$$\phi^r(x, y) = \frac{1}{\sqrt{A}} e^{ik_x x + ik_y y}$$

A: area (normalization const)

$$\Psi^r(r, t) = \sqrt{\frac{2}{L}} \frac{1}{\sqrt{A}} e^{ik_x x + ik_y y} \sin\left(\frac{n\pi}{L} z\right)$$

$$E_n = \frac{\hbar^2}{2m} \left[k_x^2 + k_y^2 + \left(\frac{n\pi}{L}\right)^2 \right]$$

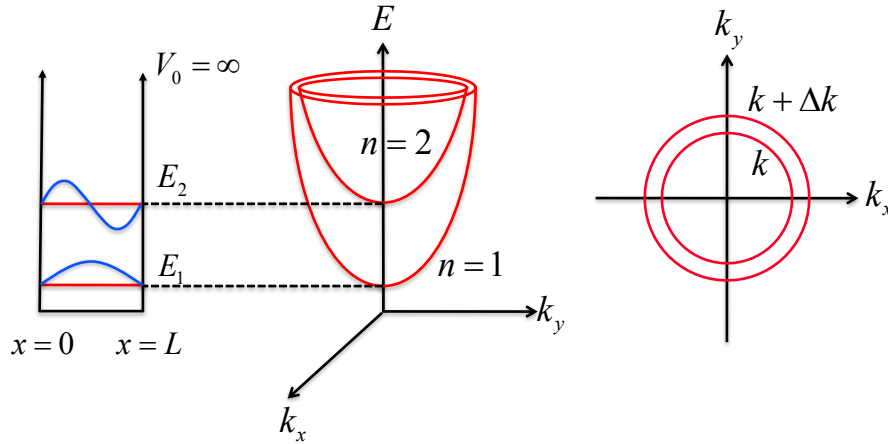
Energy quantized only in k_z direction

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2-d Density of States



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2-d Density of States

Consider the lowest band first ($n=1$):

Number of electron states between k and $k + \Delta k$
per unit volume

$$\rho_k(k)dk = \frac{2}{V} \cdot \frac{2\pi k dk}{2\pi L_x 2\pi L_y} = \frac{2}{L_z} \frac{k}{2\pi} dk$$

$$E(k) = \frac{\hbar^2}{2m_e^*} \left[k^2 + \left(\frac{\pi}{L} \right)^2 \right]$$

$$\rho_{2d}(E)dE = \rho_k(k) \frac{dk}{dE} dE = \left(\frac{2}{L_z} \frac{k}{2\pi} \right) \frac{1}{\frac{\hbar^2}{m_e^*} k} dE$$

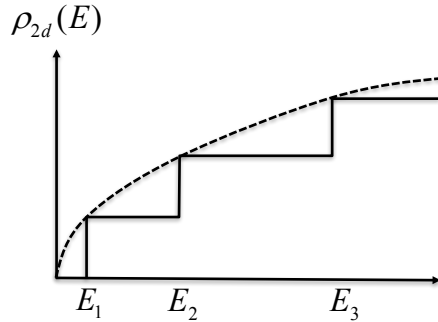
$$\rho_{2d}(E) = \frac{m_e^*}{\pi \hbar^2 L_z}$$

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2-d DOS for Multiple Energy Levels



$$\left\{ \begin{array}{ll} 0 < E < E_1 & \rho_{2d}(E) = 0 \\ E_1 < E < E_2 & \rho_{2d}(E) = \frac{m_e^*}{\pi \hbar^2 L_z} \\ E_2 < E < E_3 & \rho_{2d}(E) = \frac{2m_e^*}{\pi \hbar^2 L_z} \\ E_3 < E < E_4 & \rho_{2d}(E) = \frac{3m_e^*}{\pi \hbar^2 L_z} \end{array} \right.$$

In general

$$\rho_{2d}(E) = \frac{m_e^*}{\pi \hbar^2 L_z} \sum_{n=1}^{\infty} H(E - E_n)$$

Step Function

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2-d Electron/Hole Concentration

Electron and hole concentrations:

$$n = \int_{E_C}^{\infty} f_n(E) \rho_{e,2d}(E) dE$$

$$p = \int_{-\infty}^{E_V} f_p(E) \rho_{h,2d}(E) dE$$

At $T = 0K$, and for $E_1 < E < E_2$

$$n = (F_n - E_1) \cdot \rho_{e,2d}(E_1 < E < E_2)$$

$$n = (F_n - E_1) \frac{m_e^*}{\pi \hbar^2 L_z}$$

Example:

10-nm-wide GaAs quantum well
quasi-Fermi energy is 100 meV
above E_1

2-d electron concentration

$$m_e := 0.067 \cdot m_0$$

$$m_e = 6.104 \times 10^{-32} \text{ kg}$$

$$L_z := 10 \text{ nm}$$

$$L_z = 1 \times 10^{-8} \text{ m}$$

$$\rho_{2d} := \frac{m_e}{\pi \cdot \hbar_{\text{bar}}^2 \cdot L_z}$$

$$\rho_{2d} = 1.747 \times 10^{44} \frac{\text{s}^2}{\text{kg} \cdot \text{m}^5}$$

$$n := 100 \text{ meV} \cdot \rho_{2d}$$

$$n = 2.795 \times 10^{18} \cdot \frac{1}{\text{cm}^3}$$

$$n_s := n \cdot L_z$$

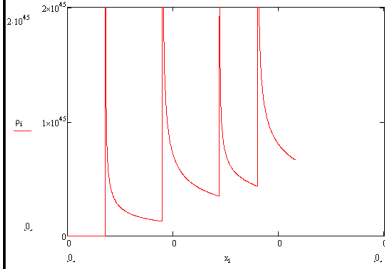
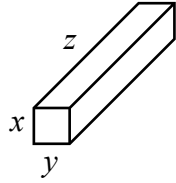
$$n_s = 2.795 \times 10^{12} \cdot \frac{1}{\text{cm}^2}$$

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1-d Density of States



$$E_{m,n}(k_z) = \frac{\hbar^2}{2m_e^*} \left(\left(\frac{m\pi}{L} \right)^2 + \left(\frac{n\pi}{L} \right)^2 + k_z^2 \right)$$

$$dE_{m,n}(k_z) = \frac{\hbar^2}{2m_e^*} 2k_z \cdot dk_z = \frac{\hbar^2 k_z}{m_e^*} dk_z$$

$$n = \frac{2}{V} \sum_{m,n} \int_{-\infty}^{\infty} \frac{dk_z}{\left(\frac{2\pi}{L_z} \right)} = \frac{2}{\pi L_x L_y} \sum_{m,n} \int_0^{\infty} dk_z$$

$$= \frac{2}{\pi L_x L_y} \sum_{m,n} \int_0^{\infty} \frac{m_e^*}{\hbar^2 k_z} dE$$

$$= \frac{1}{\pi L_x L_y} \sqrt{\frac{2m_e^*}{\hbar^2}} \sum_{m,n} \int_0^{\infty} \frac{1}{\sqrt{E - E_{mx} - E_{ny}}} dE$$

$$\rho_{1D}(E) = \frac{1}{\pi L_x L_y} \sqrt{\frac{2m_e^*}{\hbar^2}} \sum_{m,n} \frac{1}{\sqrt{E - E_{mx} - E_{ny}}}$$